EJERCICIO 3 (Jun/25)

Resuelva la ecuación por recurrencia: $u_n - u_{n-2} = \frac{1}{4}(4+3n)\cdot 2^n$

con $n \geq 2$ y condiciones iniciales $u_0 = 1$ y $u_1 = 4$

Homogenea

$$u_n - u_{n-2} = 0 \qquad x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

$$(u_n)_h = C_1 1^n + C_2 (-1)^n = C_1 + C_2 (-1)^n$$

Particular

$$(u_n)_p = (An+B)\, 2^n$$

$$u_{n-2} = \left(A(n-2)+B\right) 2^{n-2} = (An - 2A + B)\, 2^n \cdot 2^{-2} = \left(\frac{1}{4}An - \frac{1}{2}A + \frac{1}{4}B\right) 2^n$$

Sustituyo: $(An+B)\cancel{2^n} + \left(-\frac{1}{4}An + \frac{1}{2}A - \frac{1}{4}B\right)\cancel{2^n} = \left(1 + \frac{3}{4}n\right)\cancel{2^n}$

$$\frac{3}{4}An + \frac{1}{2}A + \frac{3}{4}B = \frac{3}{4}n + 1 \Rightarrow \left\{ \begin{array}{l} \frac{3}{4}A = \frac{3}{4} \Rightarrow \underline{A = 1} \\ \frac{1}{2}A + \frac{3}{4}B = 1 \Rightarrow 1/2 + \frac{3}{4}B = 1 \end{array} \right.$$

$$3/4\, B = \frac{1}{2} \Rightarrow \underline{B = \frac{2}{3}}$$

$$(u_n)_p = \left(n + \frac{2}{3}\right) 2^n$$

$$U_n = C_1 + C_2(-1)^n + \left(n+\frac{2}{3}\right)2^n$$

- Si $U_0 = 1 \Rightarrow 1 = C_1 + C_2 + \frac{2}{3} \Rightarrow C_1 + C_2 = \frac{1}{3}$
- Si $U_1 = 4 \Rightarrow 4 = C_1 - C_2 + \frac{10}{3} \Rightarrow C_1 - C_2 = \frac{2}{3}$

$$2C_1 = 1 \Rightarrow \underline{C_1 = \tfrac{1}{2}}$$

$$\tfrac{1}{2} + C_2 = \frac{1}{3} \Rightarrow \underline{C_2} = \frac{1}{3} - \frac{1}{2} = \underline{\frac{-1}{6}}$$

Solución general: $\underline{U_n = \frac{1}{2} - \frac{1}{6}(-1)^n + \left(n+\frac{2}{3}\right)2^n}$